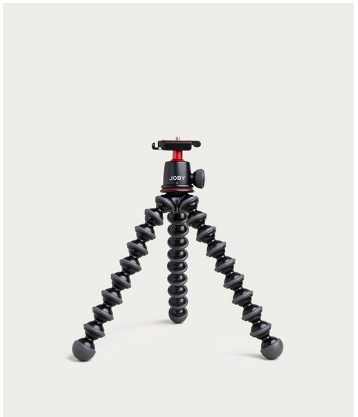
# NASA Mars 2020— analyzing the Perseverance Rover's Skycrane system.

Midterm Project Two

October 12, 2020

# Choosing a Situation to Analyze—A



## Situations that can be analyzed with 2.001

Before we begin the actual analysis portion of the project, we will start by listing some situations we chose not to analyze for various reasons. We will list these reasons before getting to the actual system we chose to analyze on the next page.

**Analysis of a "sawhorse table."** The sawhorse table could be modeled as three bars connected together taking a vertical force from the top. This system was not chosen because the presence of friction in the bottom of the structure would actually induce a bending moment in the bars which isn't ideal for analysis. The interface between the floor and the foot of a sawhorse isn't exactly a pin joint.

**Stresses on the supporting bars of a pergola.** Pergola is a really strange word but we can analyze the structure anyways. We chose not to analyze this structure of the two main members and the supporting cross-brace because we wanted to model a material that can be modeled as an elastic material, wood is not elastic. And we were unsure how to abstract the loading conditions on this part of the pergola and what forces would be on each of the members.

**Stresses and strains in a scissor lift.** The scissor lift is a classic example of an assemblage of bars taking forces together. This was not chosen because we were unsure of how to model the forces on the central pivot that connects the bars in an x-shape. We are fairly sure there will be some bending component on the members of this structure.

**Analyzing a tripod with a camera on top.** The camera tripod is another assemblage of bars that—in this case—actually can be modeled with pivot joints attaching to the floor. This system was not chosen because it seemed not useful to abstract the system into a 2-D problem. It also wasn't a very interesting problem as the forces aren't usually significant enough to cause many problems with the structure. Tripods are usually designed for stability.

# Introduction + Problem Analysis—B

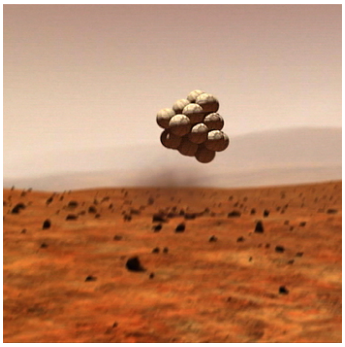


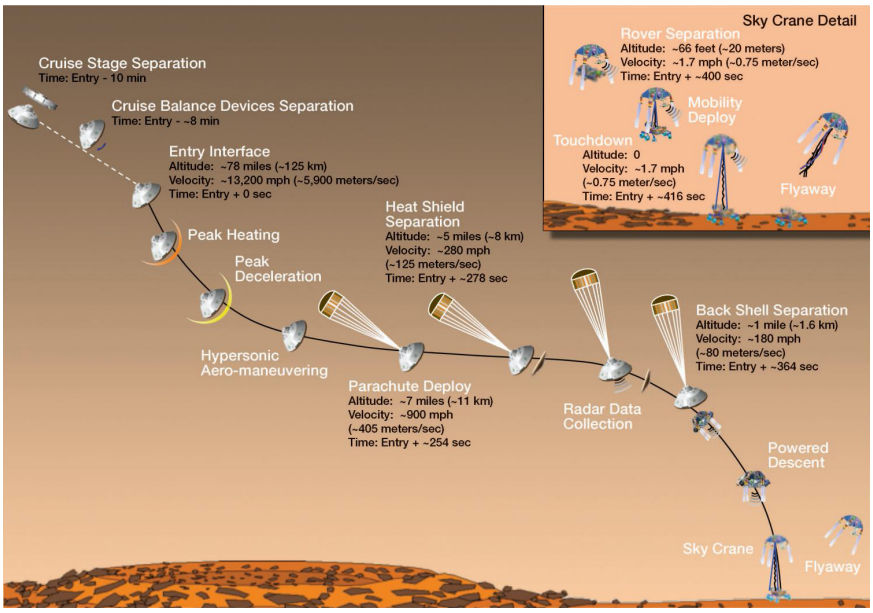
**Image 1**
Spirit rover bouncing around on the surface of MARS. (https://mars.nasa.gov/mer/mission/timeline/edl/steps/)

**Image 2**
Skycrane depositing Curiosity on the surface of Mars in contrast to the bouncy landing of Spirit in Image 1. (https://mars.nasa.gov/mars2020/timeline/landing/entry-descent-landing/)

For anyone who follows space exploration, it may be a little know fact that NASA's Mars Exploration Program found a way to inject some good news into the year 2020. The NASA Mars 2020 mission lifted-off from Cape Canaveral this past July carrying the 1000 kg Perseverance Rover and a smaller payload called the Mars Helicopter (which is a drone they're really going to fly on Mars)!

Due to factors outside of NASA's control, if we launch a rover to another planet we have to—turns out—figure out how to actually *land* it on the surface of the planet (so getting it into space is just step one). With the Perseverance Rover this proved to be a tricky little task. Most of NASA's previous spacecraft only weighed around 100kgs, that's 10x less than the new rover. Which means they could slow themselves down to land with a simple parachute and a beach-ball. (www.youtube.com/watch?v=6t3IARmIdOI see the last 3 minutes of this video).

**Turns out the people at NASA are very smart and found a slightly insane—yet brilliant—solution to this problem.** They called it the Skycrane, and because they're much better at explaining it than us, here's a video and a picture (www.youtube.com/watch?v=p1WX0CATyn8):




This terrifying process is called the **Entry, Descent, and Landing (EDL)** stages of the mission. And you only have one shot to get it right. We're interested in analyzing the frame of the Mars Skycrane module which can be modeled as an assemblage of bars. Read more at the NASA-Mars website: mars.nasa.gov

# Modeling + Abstraction—B

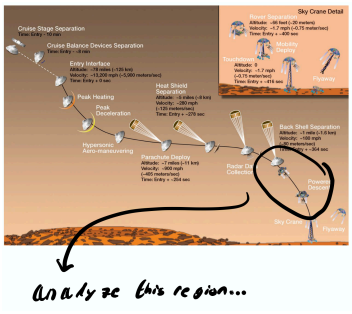



**Image 3**
Region of interest of the problem—we're only analyzing the powered-descent section of the EDL.

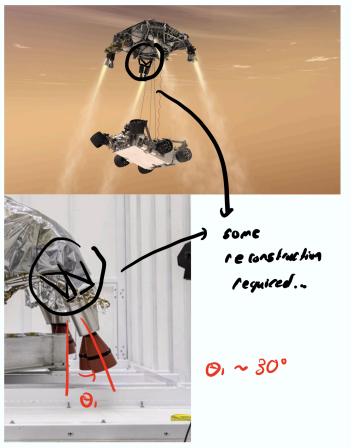


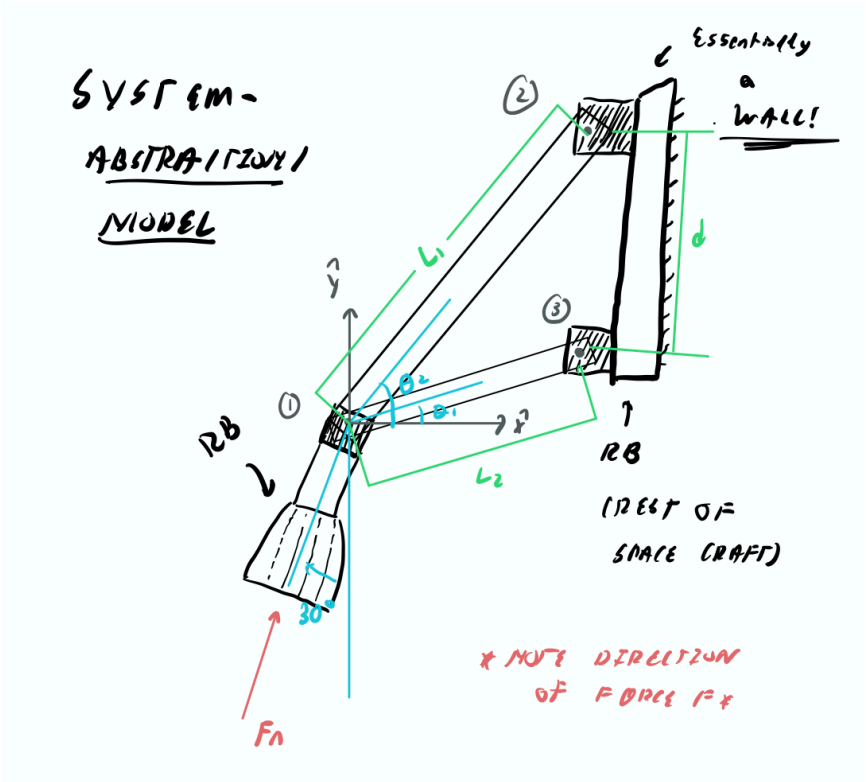

**Image 4**
Section of the Skycrane we are analyzing. The frame that serves as the interface between the retrorockets and the rest of the vehicle.

## Simplifying the problem

Of course, with this incredibly complex problem we don't plan to analyze all of its aspects. So we shall choose one sub-assembly inside the complex Skycrane structure and one region of operation of that subassembly to analyze. We are choosing to analyze the frame that serves as the interface between the retrorockets—any rocket used to slow a vehicle or a rocket fired in a direction *opposing* a vehicle's motion—and the rest of the spacecraft. **We will only analyze the frame in the powered-descent region of the flight trajectory.**

## Modeling assumptions

Before we get to any proper assumptions about the system, regions of operation and etc, we want to first abstract the frame into a model that can be used to analyze the structure of the Skycrane under the loads it will be experiencing.




After carefully examining images of the structure, we came up with the above drawing of the system which is comprised of two bars attached together via a pin to the retrorocket on the left, and

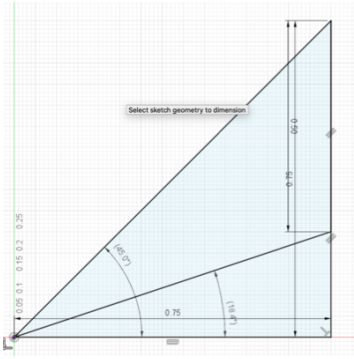



**Image 5**
Geometry assumptions of the Skycrane system, this was developed using pure convenience as we couldn't find the data on the design.

then attached by pin-joints to the wall on the right. We are going to assume this is the geometry of the system and we are going to exert the retrorocket's thrust force on the Pin 1 joint at the angle we think it is firing at, we will then calculate some system parameters we will describe later. For now, there are some significant assumptions we are making that we must state first.

**Assumptions relating to the geometry of the structure.** ONE—the geometry of the system we are analyzing is that of which is described in the image on the previous page and in Image 5 to the left. We are assuming these things for pure convenience, and because this project is more focused on the ability to model as opposed to actually designing the Skycrane for NASA. TWO—we have abstracted the problem to two-dimensions where there are only two bars supporting a single rocket firing at a 30° angle from the vertical. Again, we are making these assumptions purely for convenience NOT FOR ACCURACY. Realistically, this structure should be analyzed as a 3D assemblage of bars with both rockets, with the correct dimensions and angles. THREE—we have also assumed that the joints are pin-joints—we cannot tell from the image of the structure itself—but it is very possible that they are not pin-joints and they are welded joints. This assumption is made for convenience and analysis purposes so we may use 2.001 techniques to analyze the bar as a two-force member.

**Note 1**
If you would like to know why these assumptions are very inaccurate, do some research on the Skycrane, EDL, and how rockets work. Think! We're using fuel (mass can't be constant), and the rockets are also used to keep the craft upright not just slow it down.

**Assumptions related to the loading of the structure.** FOUR—we will make the hilariously unreasonable assumption that the mass of the Skycrane stays constant through the landing. FIVE—we will make the other ridiculous assumption that the rocket force stays constant through the trajectory. SIX—we will make the assumption that the bars in the structure are stuck into a "wall" on the right—through pin joints—also a questionable assumption.

**Assumptions related to EDL.** SEVEN—we will assume the craft—in the region of operation in question—will descend at an angle of entry of 0 deg. The reasons for this assumption will become clear later. Again this is done for simplicity and not accuracy. EIGHT—we will assume acceleration is constant through the trajectory.

**IF YOU ARE CONFUSED—we understand these assumptions may be a little strange to you. Here's what you need to know: The point of all of that is really just for us to be able to say, "let's analyze this system using the drawing we made on the previous page and exert some force at some angle." NOT to create an accurate model of the system. If you are confused about the assumptions that's OK, come to OH or just focus on how we did the math below. We will explain the assumptions in more detail as we go through the calculation.**

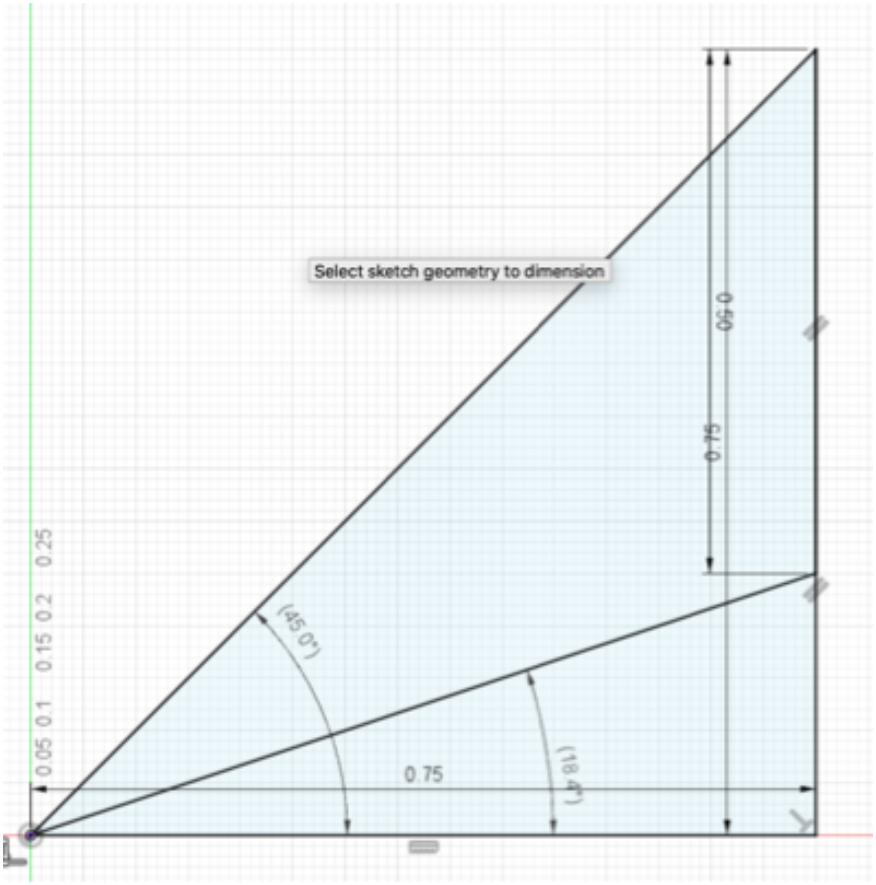
## Background information

**Note 2**
Properties of Ti 6-4:
Y = 880 MPa (tensile yield strength)
E = 113.8 GPa

Of course, we will need some basic information to solve the problem we will create below. This includes things like the geometry of the bars—**which we will assume to be solid-cylindrical bars with radius "r1" and "r2."** We also want to know what materials to make the bars out of. **We will choose aircraft grade Titanium—Ti 6-4—because of its strength to weight ratio.** The material properties of Titanium required for this problem are listed to the left, taken from Matweb.

We will also give students enough information to determine the lengths of the bar, the angles, and other geometric parameters.

**Note 3**
Random CAD drawing we're using to determine the geometry of the structure.



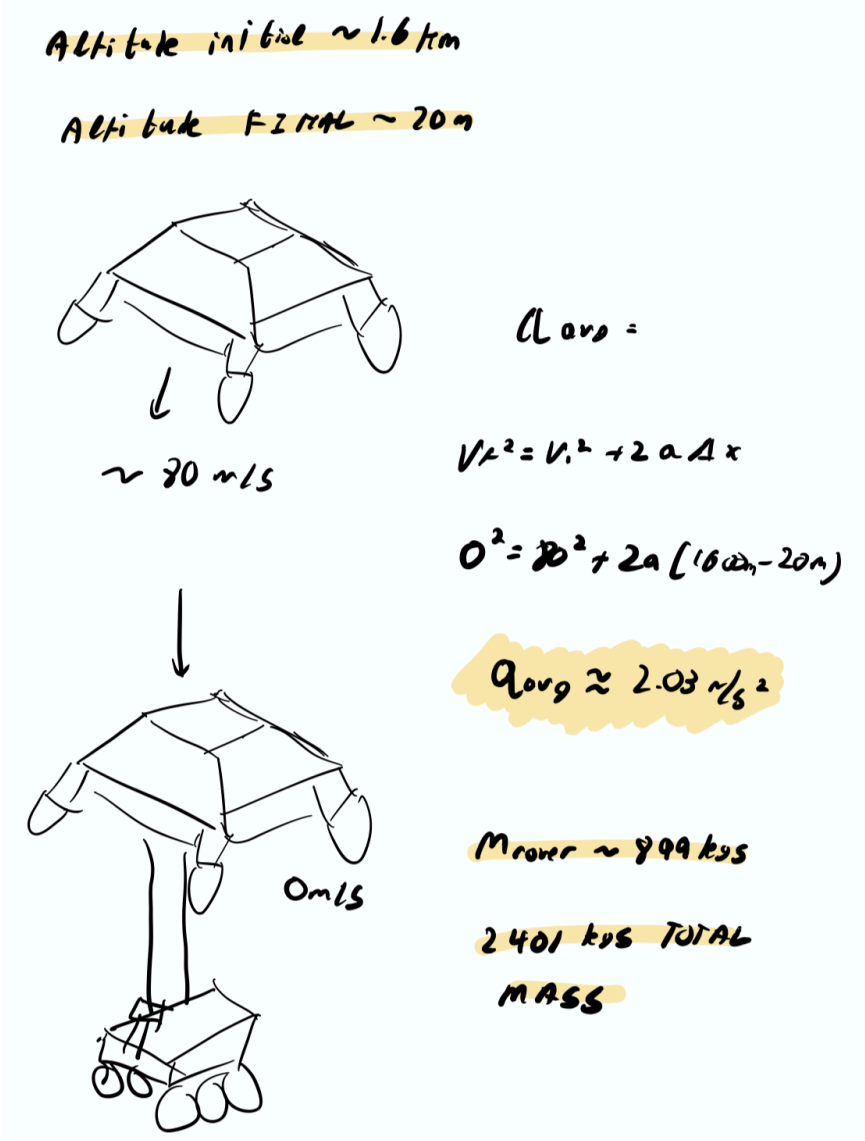

These are the dimensions given, with the lengths in meters and the angles provided. We're assuming the top-bar is the hypotenuse of a right-triangle with side lengths of 0.75m and 0.75m. We're saying the same for the bottom bar but the side lengths are 0.75m and 0.25m. Theta1 = 18.4°, and Theta2 = 45°. The height between the two pin-joints on the right is 0.5m.

**The final piece of information we need is the rocket force on the structure.** We are not going to make students calculate this because—even though it isn't that hard to figure out with a basic

FBD—it's a little tedious and introduces the concept of a virtual force. That calculation is presented here for those who are interested:

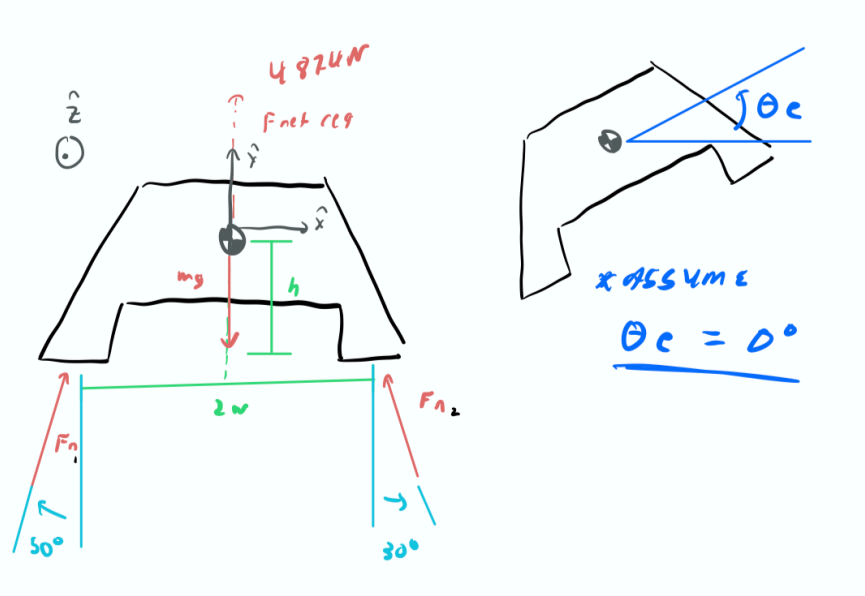
## Calculation of Rocket-Force

First, let's figure out the acceleration required to make this system slow down from 80m/s to 0m/s as it descends from 1.6km to 20m as indicated in the video and in the diagram on page 3. We'll use some very simple kinematics.

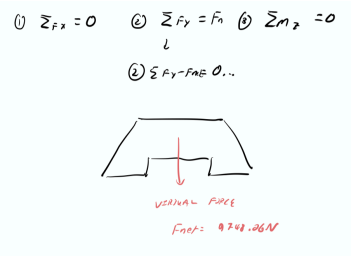



That wasn't so bad right? We now know what the acceleration of the body has to be to achieve the desired behavior. Now let's

make a FBD of this system to see how the system is loaded to find the rocket forces.



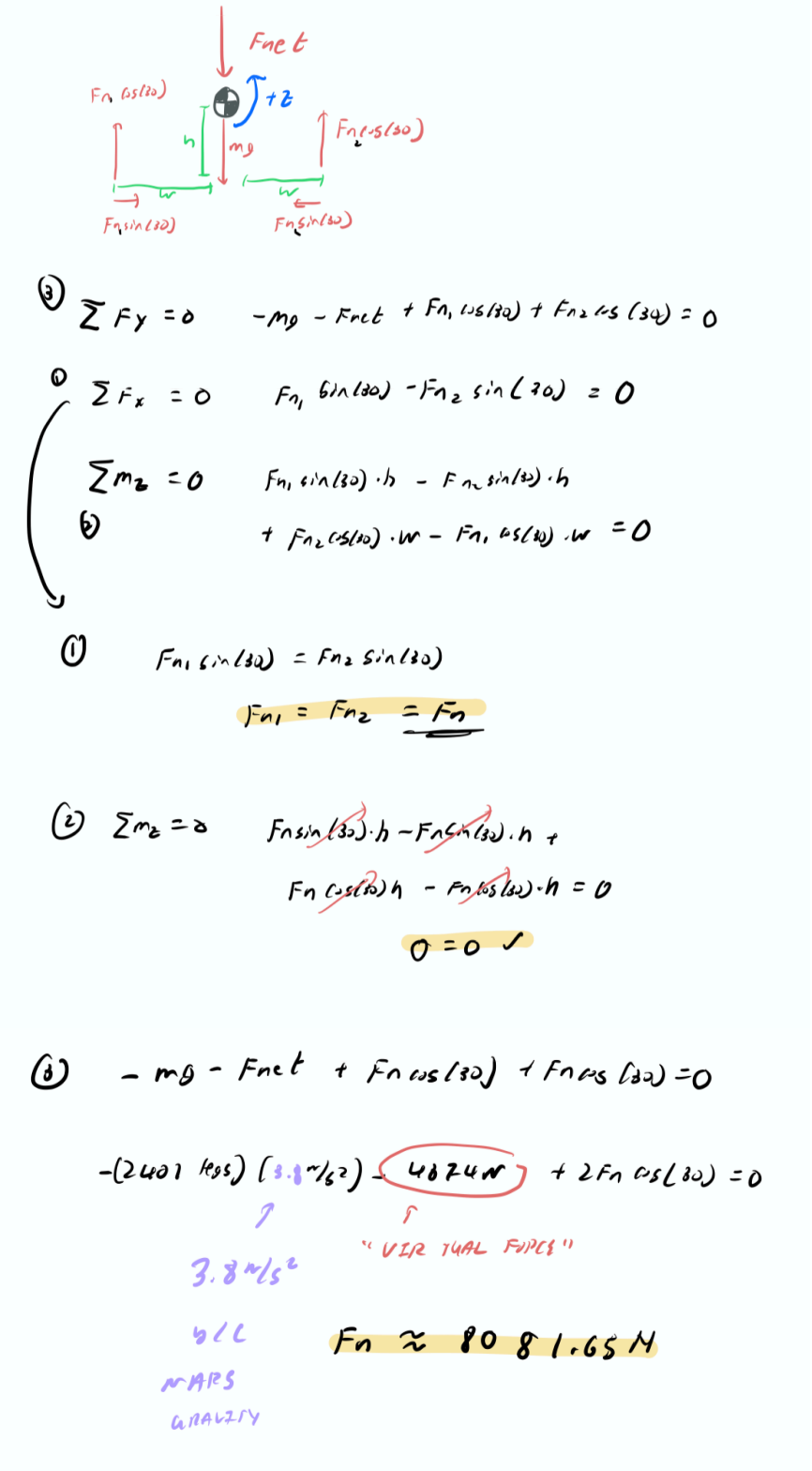

So we know there's a force of gravity downwards, we know there are—let's say TWO rockets—firing in the upwards direction to slow the system down at an angle of 30°. And these must sum to a net force of F=m*a—the "a" we just calculated—in the upwards y-direction.



**Note 4**
Placing the virtual force at the COM of the structure.

Let us briefly explain what virtual force is. As you know, this system is NOT static in that it is accelerating in the upwards direction, has a velocity in the downwards direction, and is slowing down right? To solve this for Fn2 and Fn1, we could just say sum of the forces-y needs to be F=ma. But we could also make the sum of the forces-y = 0 and make this problem a statics problem by saying "okay, to make this problem static we'll place a downwards force on the system EQUAL to F=ma so the sum of the forces-y = 0, if we know what the mass and the acceleration are." Since that force doesn't actually exist, and we're just doing that so the sum of the forces can be zero—to make the problem static—we call it a virtual force. **In this case, you could also just ignore the concept of virtual force and simply say the sum of the forces-y is F=ma.**

The calculation for the final rocket force is done on the next page. We simply used the sum of forces = 0—if we use a virtual force—and that the sum of the moments is 0.

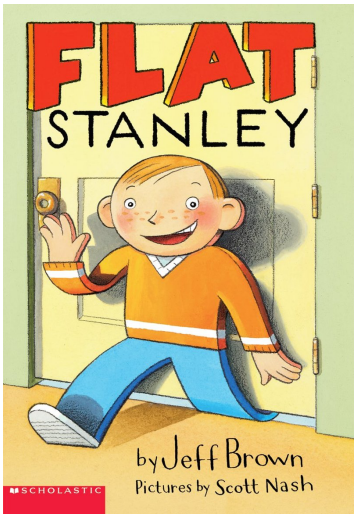



**Note 5**
Sum of the forces x and y must equal zero on this rigid body. Sum of the moments z must equal zero.

**Note 6**
Solve the force and the torque equations, find that the lengths cancel due to symmetry.

**Note 7**
And we have our rocket force!
**NOTE THAT g=3.8m/s^2 BECAUSE WE ARE ON MARS.**

This gives us a final rocket force, assuming two rockets, of 8081.65N per rocket. **Yes, there are four rockets, but just**




**Note 8**
Flat-Stanley. (Image from Scholastic)

**assume for argument sake we're doing a 2D Skycrane and we're landing Flat-Stanley on Mars or something.**

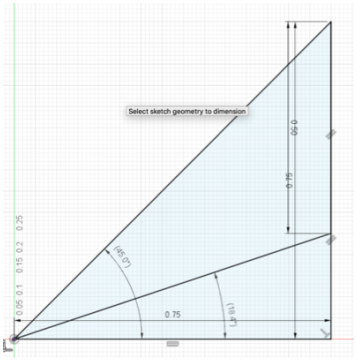
We're now ready for the problem statement and to do some engineering on the bars in this structure.

**Also, please don't think your abstraction and problem statement needs to be this long. This problem just required a lot of preface. Also, our drawings are big.**

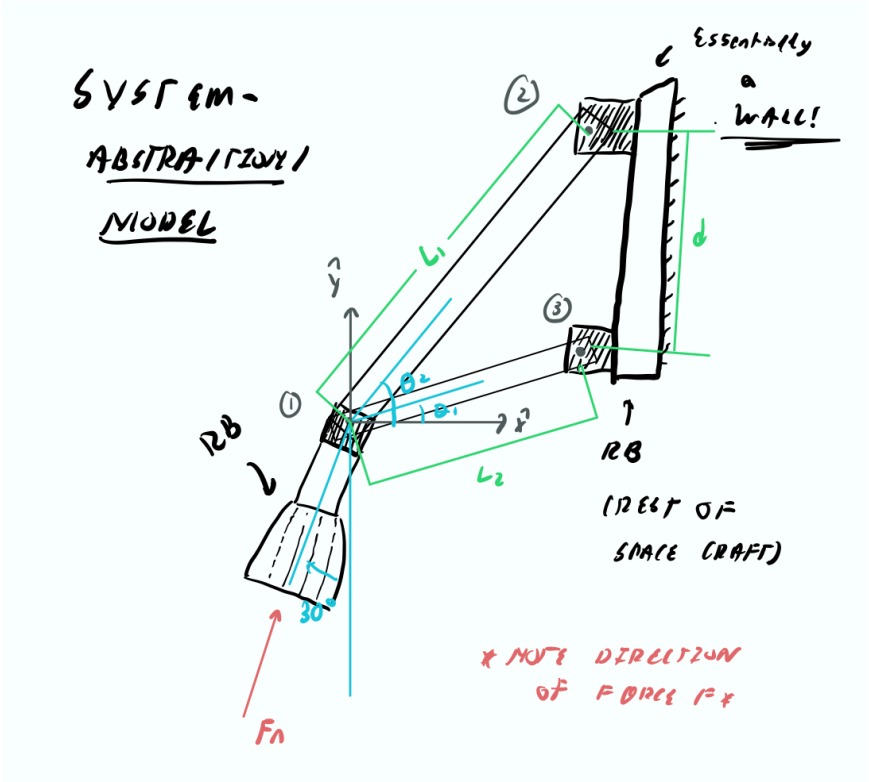
# Problem Statement—C

## 2.001 MITx-style question

Suppose we have two Elastic Perfectly-Plastic bars organized in the assembly below, with a force Fn being applied on the right at a 30° angle to the vertical. All joints are pin joints, the bars have solid-circular cross sections with radius “r1” for bar “L1” and “r2” for bar “L2.” The bars are made of Ti 6-4 with E=113.8 GPa and Y=880MPa.




**Note 9**
Use the diagram above to determine L1, L2, Theta1, and Theta2.




**Problem 1.** Suppose Fn = 8081.65N. Using the above information and the diagram, calculate the following:

- Calculate the reaction forces in the x-direction at Pins 2 and 3 by putting the pins and the entire structure in equilibrium.
- Calculate N1 and N2 of the bars.

**Problem 2.** Now we want to design the radius of the bars such that the structure survives:

- Calculate the radii for each bar such that it doesn’t yield.
- **For bonus points,** account for buckling, and size the radii such that the bars displace proportional to their lengths.

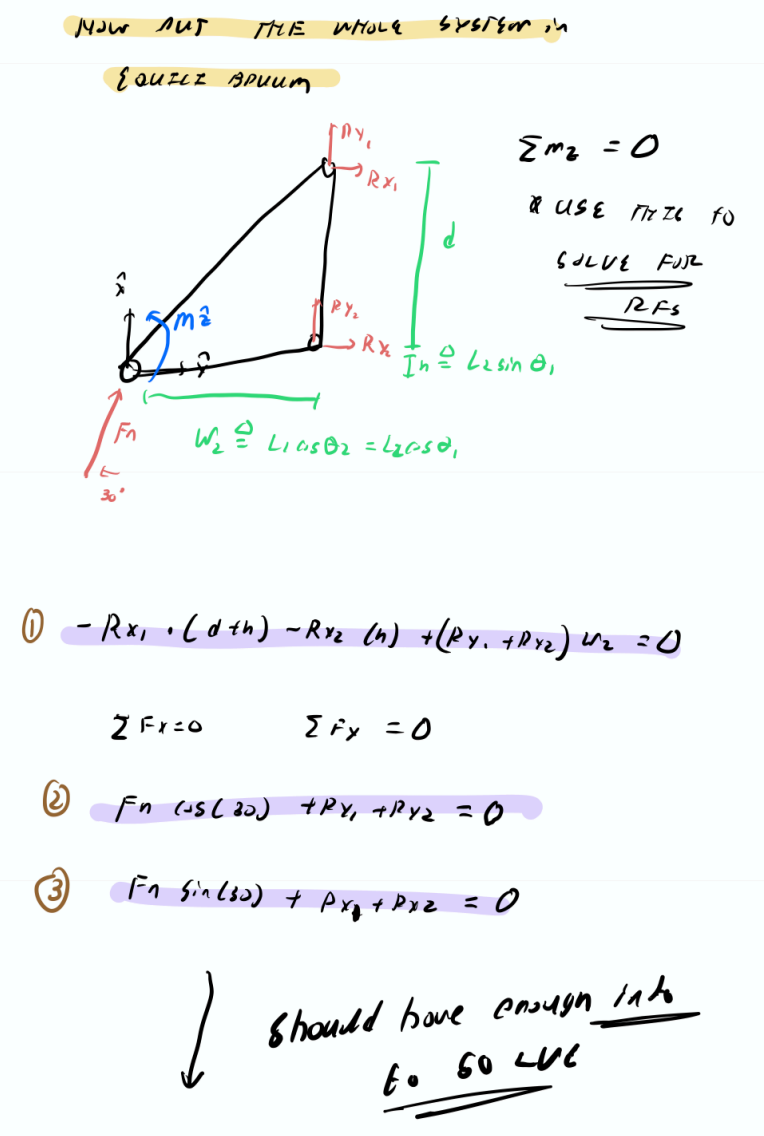
**Note 10**
Please don’t cry if you don’t understand the second part of problem two. It’s ok, we don’t expect you to, it’s just for fun + if you’re feeling curious!

# Calculations—D

## Calculate the reaction forces

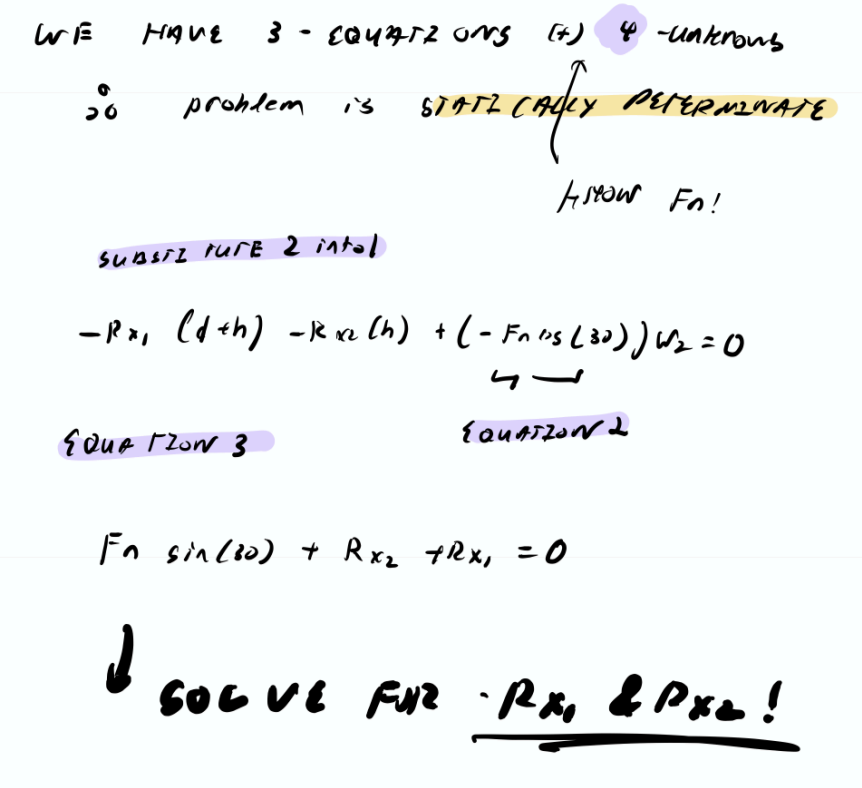
To calculate the reaction forces at the mounting points of the system, we're going utilize an isolation bubble that encloses the entire structure.

**Note 11**
Model the entire structure as a single rigid body.




We use the sum of the forces-x, sum of the forces-y, and sum of the moments-z on the rigid body all have equal zero. For this calculation we simply assume everything is one rigid body.

The next step is to actually solve the equations we came up with to find the reaction forces at the pins.




$R_{x_2} = 4437\ N$

$R_{x_1} = -8478\ N$

**Note 12**
And then we got the reaction forces Rx1 and Rx2 as above.

To simplify the calculate we use MATLAB to solve the system of equations:

```
syms Rx2 Rx1

Fn = 8081.65
theta = 30*pi/180
eq1 = -0.75*Rx1-0.25*Rx2+(-Fn*cos(theta))*0.75 == 0;

Rx1 = solve(eq1, Rx1)
eq2 = Fn*sin(theta)+Rx2+Rx1 == 0
Rx2s = double(solve(eq2, Rx2))
Rx1s = double(subs(Rx1, [Rx2], [Rx2s]))
```

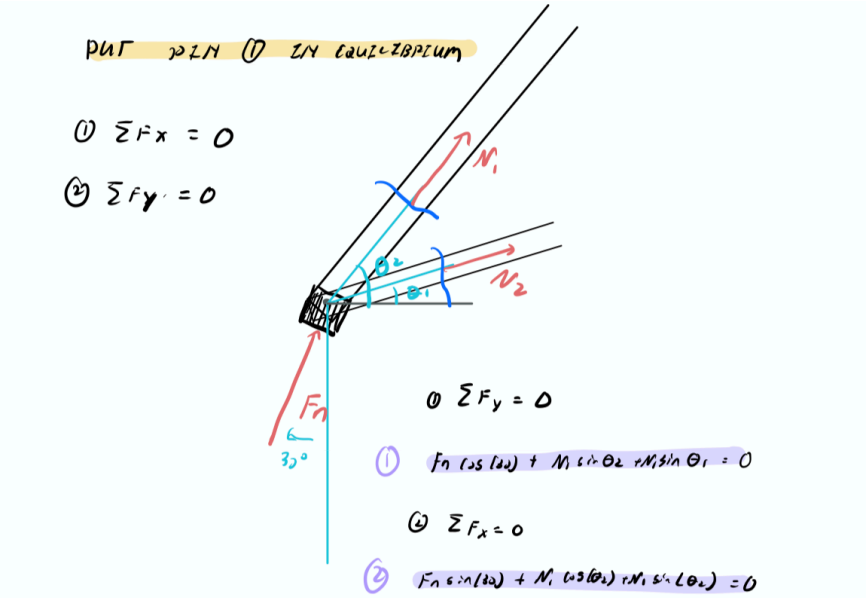
```
Rx2s =

   4.4371e+03


Rx1s =

  -8.4780e+03
```

We were only asked to solve for the reaction forces in the x-direction because that's all that's needed to calculate N1 and N2 in the bars as we will now demonstrate.

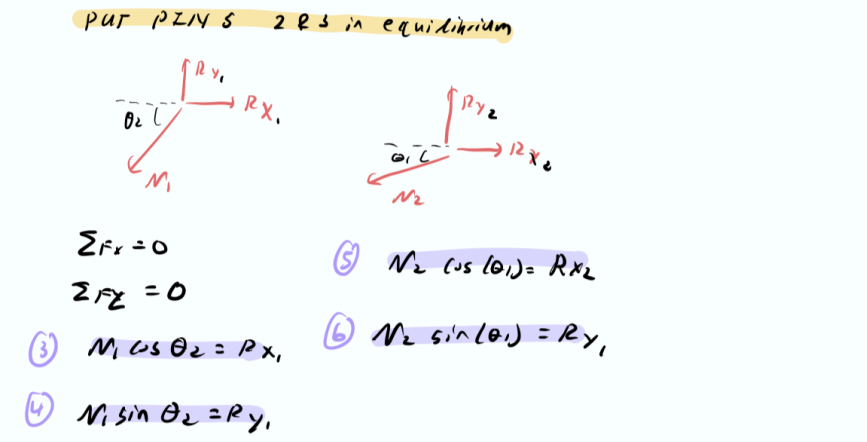
## Calculate the N1 and N2 in the bars

To calculate the axial forces in the bars, we want to place the pins in equilibrium. So this means—instead of modeling the structure as a single rigid body—we'll model is as a bunch of rigid bodies connected by pins.

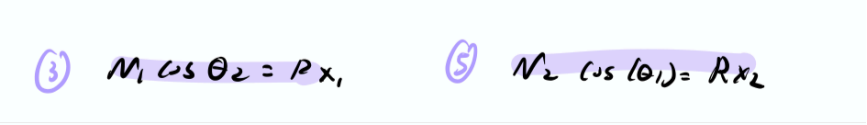
**Note 13**
Put pin 1 in equilibrium as an equation to have.



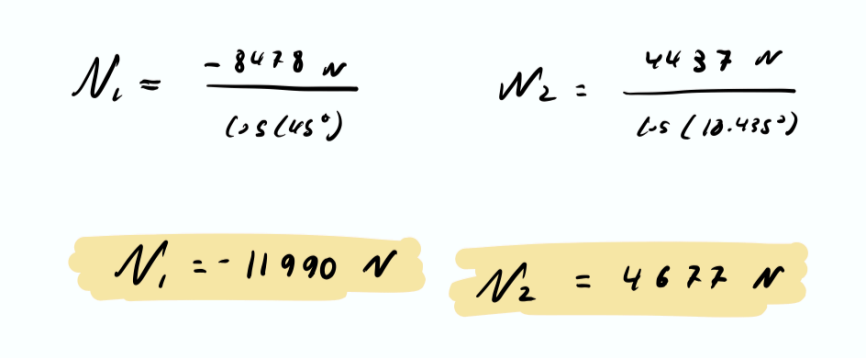

**Note 14**
Put pin 2 and 3 in equilibrium to get a relation between N and the Rx at that pin specifically.




Now we'll use what we found in the previous calculation to calculate the axial forces in the bars.




**Note 15**
Then use the calculated reaction forces to solve for the axial forces in the bar.

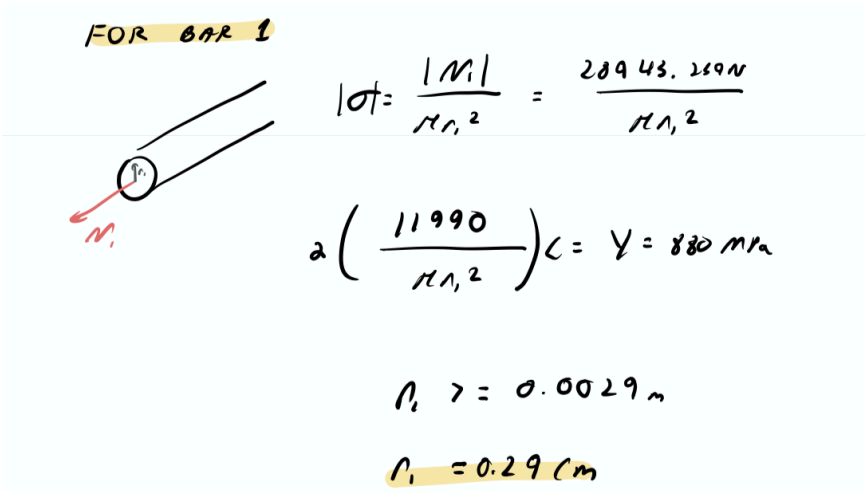


With these forces we know the top bar is in compression, and the bottom bar is in tension.

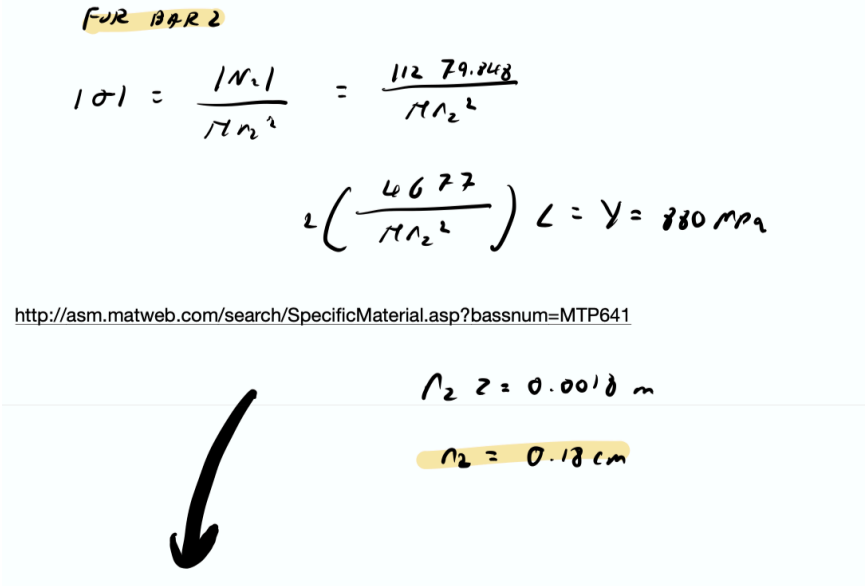
## Calculate “r1” and “r2” so the bars don’t yield

We’re going to assume a safety factor of 2 in this calculation to keep the bars light. The calculation is fairly straight forward now that we know N1 and N2!

**Note 16**
Using the yield strength of titanium, calculate the r so the bar doesn’t yield. For bar 1.

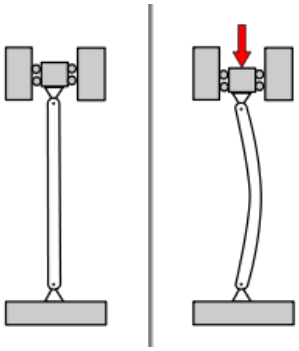



**Note 17**
Using the yield strength of titanium, calculate the r so the bar doesn’t yield. For bar 2.




What this calculation is telling us is, because of the strength of Titanium alloy, these bars can be <1cm in diameter! **You might think that’s ridiculous and that would never work. And you’d be right. There’s something else we need to account for which we shall do in the next part.**

# Accounting for buckling and more uniform elongation



**Note 18**
Buckling of a bar, image from Wikipedia.

**A little bit of preface.** It turns out that slender bars in axial loading have another point of failure called buckling. This occurs when the bar isn't being loaded enough for it to technically yield axially, but the stiffness of the component in question cannot take the loads being applied. Take a look a this image from wikipedia, below is some information on buckling:

D9_1: Buckling Of Slender Columns

Structural instabilities and buckling are very dangerous modes of failure because they can lead to sudden catastrophic structural collapse with very little forewarning.

(Euler) Buckling of slender columns under (compressive) axial loading is the most easily mathematically treatable of these conditions.

We briefly discussed the case of a round slender column, of length $L$ and constant flexural section rigidity $EI$, fixed at one end and loaded by a compressive axial load $P$ at its free end.

The critical buckling load for this structure is

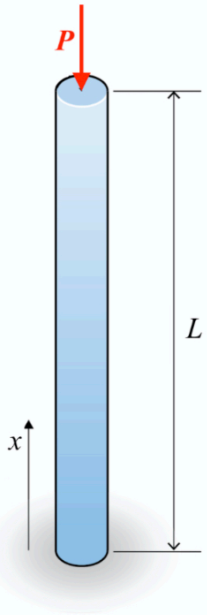
$$P_c = \frac{\pi^2 EI}{(2L)^2}$$

The more general form of the expression for the critical buckling load for a slender column of length $L$ and constant flexural section rigidity is:

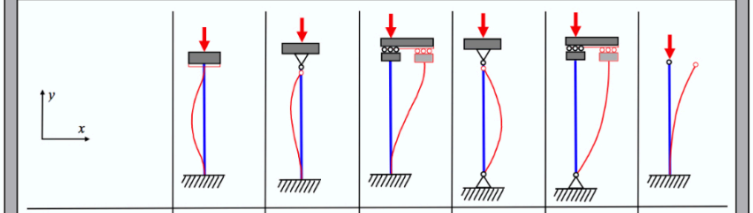
$$P_c = \frac{\pi^2 EI_{\min}}{(KL)^2}$$

where $I_{\min}$ is the weakest moment of inertia of the cross section, and the length correction factor, $K$, depends on the constraint conditions at the column supports as shown in the table below.

Note that the values in the table are theoretical values and more conservative estimates should be used in practical applications where small load eccentricity or relaxed constraints at the supports can result in drastically diminished levels of the critical buckling loads.




*Support Conditions, Buckled Shapes and Length Factors for Axially Loaded Columns*




| *Effective Length Factor, K* | 0.5 | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
|---|---|---|---|---|---|---|
| *End Constraint* | | | | | | |
| *Prevents* | $u_x\ u_y\ \varphi_z$ | $u_x\ u_y$ | $u_x\ \varphi_z$ | $u_x$ | $\varphi_z$ | / |

All we're trying to say is there's some critical load where this column will fail when loaded axially and it's based on the formula above. **Assuming it doesn't yield first.**

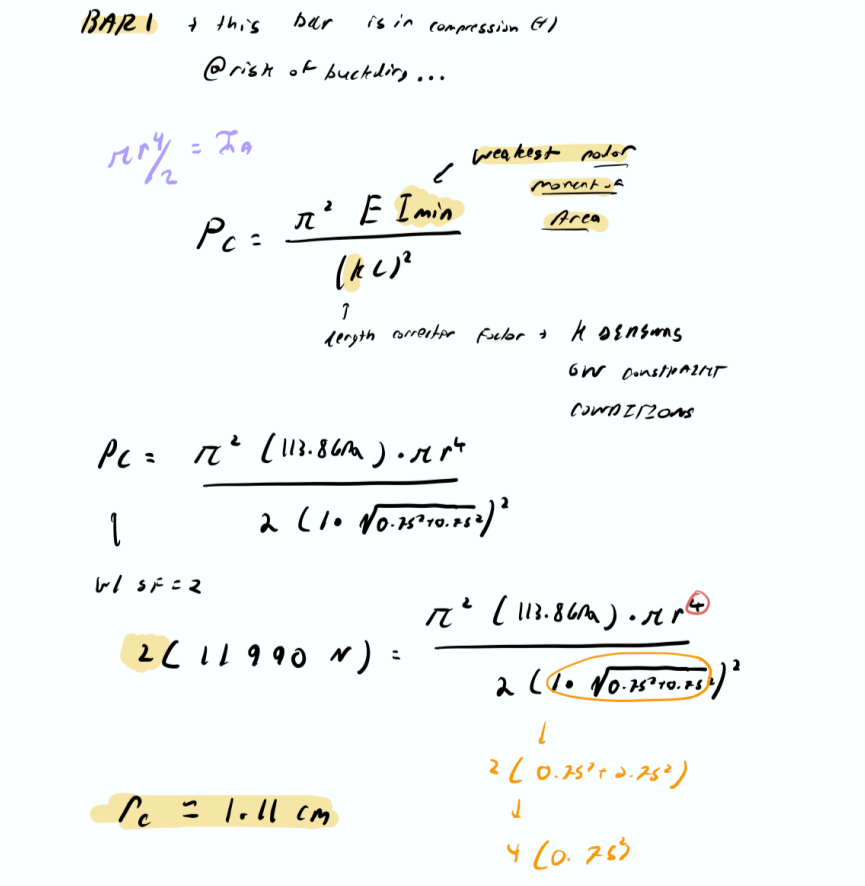
**Note 19**
The moment of inertia is used to help describe the stiffness of a cross-section. We will go over it later in the class. (https://en.wikipedia.org/wiki/Second_moment_of_area)

The only two terms in that equation that might not be familiar to you are “k” the length correction factor, and “I” the moment of inertia. The moment of inertia of a circular cross section is given by **(pi*r^4)/2**, and the length correction factor is based on how the ends are fixed. In our case we have pin joins on both ends so **k=1.0**, as shown in the table above. **Note, bars in compression will buckle, bars in tension will not.** (https://www.engineersedge.com/polar-moment-inertia.htm for polar moment of inertia)

**Note 21**
Write down the formula for critical buckling load.

**Note 22**
Substitute in the known values for the length of the bar, and the formula for the MOI. Also add a 2x safety factor for buckling load.

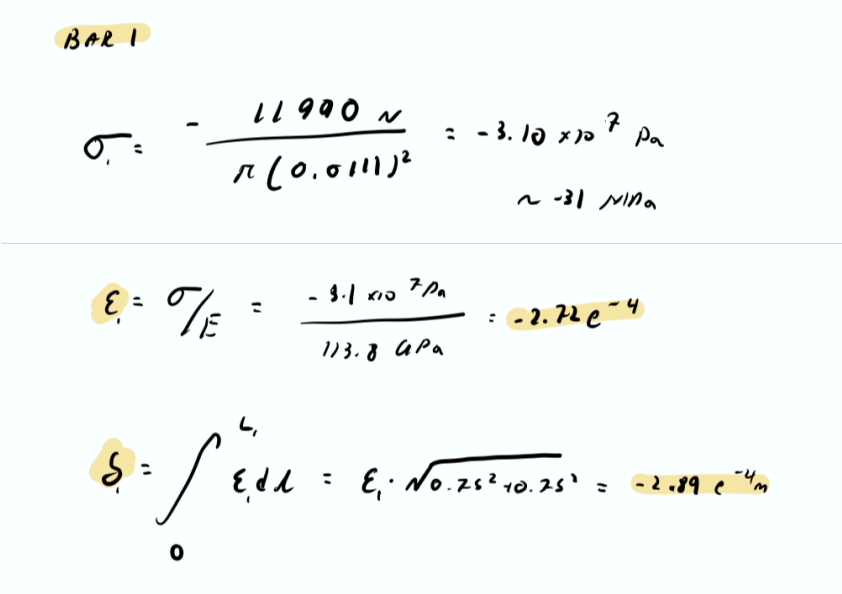
**Note 23**
Calculate the critical radius.



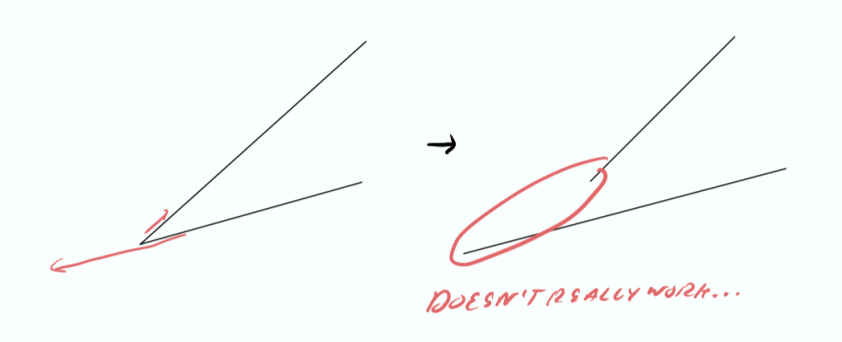

The buckling calculation is actually relatively straight-forward once you know the moment of inertia formula and the value of “k.” What we get is that the bar’s radius—so that it doesn’t buckle with a safety factor of 2—actually should be around 1.11cm. So that’s around 1 inch in diameter!

**The next step is to find the stress, strain, and elongation in bar 1 with this radius.** This is a calculation that should be relatively familiar and is shown on the next page.

**Note 24**
Calculate the stress, strain, and elongation of bar 1.

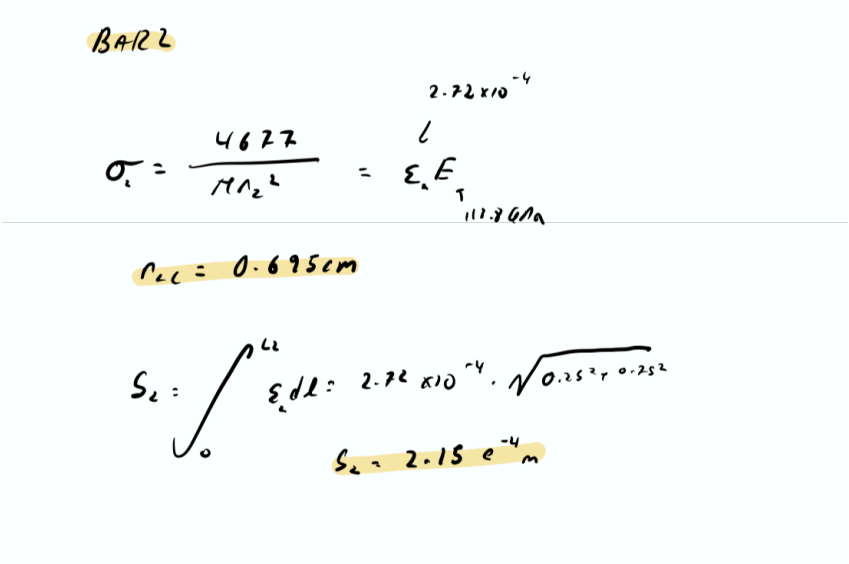



**Now there's one last set of things we want to be able to do.** We could use this radius of bar 1 to calculate the displacement, and we could use the r2=0.18cm radius calculated in the previous part to find that displacement and then find the total displacement of the pin 1. But the engineers are NASA are concerned that if the bars displacements are too different from each other this could cause issues in the structure. Remember that the **pin displacement method is really only valid for small displacements:**



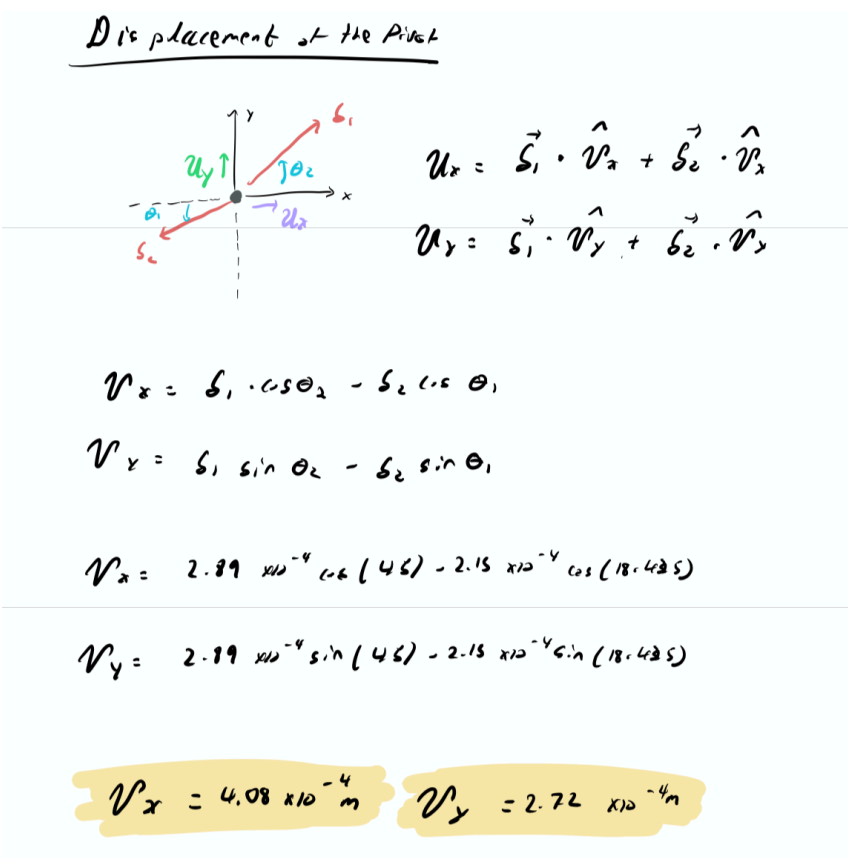

So the NASA engineers have decided the elongation of both bars should be of similar magnitudes. So they decide, what if they set the elongation per unit length for both bars to be the same? That is, what if they design the bar 2 to have the same magnitude of strain as bar 1!

**Note 24**
Assume the same magnitude of strain, calculate the radius of the bar such that the strains match (in magnitude not direction).



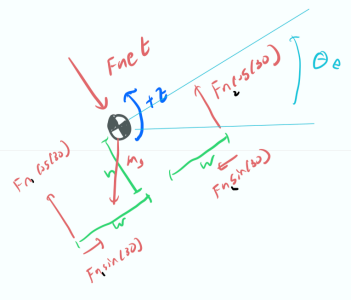

So now that we know the radius of both bars and their respective elongations, what we can do is determine the total displacement of pin 1 in the structure:

**Note 25**
Some relatively not-painful vector math leads you to the displacement of the pin.




So now we've sized our bars, and we know the displacement of the pin joint when the system is loaded w/ a given rocket force.

# Analysis + Last Thoughts—E




**Note 26**
Angle of entry changes the FBD of the system significantly.

So now we should talk about why this calculation likely won't land us a job at the Jet Propulsion Lab just yet. Remember back to the assumptions where we stated most of the assumptions we were making were for simplicity and convenience, not accuracy. We're going through the assumptions now to determine how we could make our model better in the future with more knowledge.

**Assumptions related to geometry.** One of the reason we can't say this calculation accurately models the loads on the bars in the Skycrane is because we literally guessed the geometry of the system from a picture and came up with dimensions that seemed reasonable—but are based in no-way on any data NASA has released—to make the calculations easier and simple. So even if the rocket force was accurate, NASA wouldn't be using these radii in their system.

**Assumptions related to EDL.** We also assumed that the angle of entry is 0° to the horizontal and there were only two rocket engines for the 2D simplification. Now let's not talk about the two engines vs four engines or anything, just the simple fact that the angle of entry for most spacecraft is very much larger than zero so the heat-shield may protect the craft from burning up while entering the atmosphere. Why does this matter? Well in the rocket force calculation section, we drew a FBD that would significantly change with any change in angle of entry. See the diagram to the left. What we would do in this case is take the maximum of the rocket forces and use that in the calculation.

**The most important problem with this calculation.** Is not the assumptions related to the angle of entry or the geometry of the Skycrane—like what if we did enter at 0° and that was the geometry—that's all technically possible. No, the biggest problem with the above calculation is the fact that we assumed bars 1 and 2 are stuck into a wall on the right hand side, and assumed applying the rocket force F is an appropriate way to analyze this system. For all intents and purposes it isn't really. Gravity isn't applied as a point load at the COM, it's distributed across the structure and there's a whole rest of the structure that should help take these loads on the Skycrane as well which we are neglecting. The best method to analyze this structure is likely some form of computer simulation.

**Best of luck on project 2!** And happy modeling everyone :)